NOTATION

P_+, larger of the pressures separated by the window; P_-, smaller of the separated pressures; P_0, total pressure of the working gas at the input to the nozzle receiver of the window stage; G, gas flow; G_N, normalized flow; Re, Reynolds number. The indices 1 and 2 denote the number of the window stages.

LITERATURE CITED

1. R. N. Guile and W. E. Hilding, "Investigation of a free-vortex aerodynamic window," AIAA Paper No. 75-122 (1975).
2. A. A. Gilerson, V. I. Panchenko, V. T. Rafikov, et al., Uchebn. Zap. TsAGI, 21, No. 4, 104-108 (1990).
3. V. M. Ievlev et al., Izv. Akad. Nauk SSSR, Ser. Tekh. Nauk, 13, No. 3, 52-56 (1977).
4. B. A. Ivanov et al., Pribl. Tekh. Eksp., No. 4, 153-154 (1989).
5. V. V. Shelomovskii, Uchebn. Zap. TsAGI, 13, No. 1, 102-106 (1982).

MASS TRANSFER FOR BAROMEMBRANE SEPARATION OF SOLUTIONS

V. V. Beloborodov, O. V. Konstantinova,
and É. F. Rusar

UDC 541.12.012.5:621.039.341.22

A mathematical model is obtained for transient mass transfer during the baromembrane separation of liquids for a flat channel with a semipermeable wall. Hydrated sunflower oil is used as an example to show that the theoretical and experimental time-dependences for the content of the retained material in the concentrate differ insignificantly.

Published efforts on the mathematical modeling of material separation with sempermeable membranes can be divided into two groups. In the first [1-4] the transient mass transfer problem is solved for an unmixed medium in a batch cell, which usually is used in the laboratory. In the second group [5-8], the problem is for stationary mass transfer for laminar motion of the medium in a channel with a semipermeable wall.

Moreover, in [1-4], the concentration at the separation surface (membrane—polarized layer, polarized layer—solution) is taken as a constant, while it actually depends on time. Obviously there is interest in solving the transient problem of mass transfer for an arbitrary motion of the medium (for any value of the convective diffusion coefficient) and for time-varying concentrations at the separation surface.

The separation of materials with the use of semipermeable membranes (ultrafilters, microfilters, and reverse osmosis) can be represented in the following fashion (Fig. 1):

1) The separated material is concentrated at the plane $x = R$ becuase of the removal of the permeate through the membrane;

2) The material enters a flow of concentrate through the plane $x = 0$ by convective diffusion;

3) There is molecular diffusion in the diffusion boundary layer ($0 \leq x \leq R$) of material from the membrane to the flow (concentrate) side;

4) The liquid medium (in the general case a mixture of the permeate and the separated material) filters through the membrane according to Darcy's law.

What has been said allows the problem to be described in the form

$$\frac{\partial y(x, \tau)}{\partial \tau} = D \frac{\partial^2 y(x, \tau)}{\partial x^2} \quad (\tau > 0,\ 0 < x < R), \tag{1}$$

Scientific Production Union of the Oil and Fat Industry, St. Petersburg. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 61, No. 6, pp. 953-957, December, 1991. Original article submitted February 26, 1991.

0022-0841/91/6106-1496 $12.50

$$y(x,0)=y_0, \tag{2}$$

$$-D\frac{\partial y(0,\tau)}{\partial x}-\beta\left[(y_0-f\tau)-(y_0-g\tau)\right]=0, \tag{3}$$

$$-D\frac{\partial y(R,\tau)}{\partial x}+\frac{kP}{\delta}=-D\frac{\partial y(0,\tau)}{\partial x}. \tag{4}$$

If the membrane is completely impermeable to the retained material (k → 0, one-sided isolation of the boundary layer mass), the boundary condition (4) is transformed to

$$\frac{\partial y(R,\tau)}{\partial x}=\frac{\partial y(0,\tau)}{\partial x}. \tag{5}$$

We require the concentration profile (the average concentration) at the boundary concentration layer, where partial or complete mass isolation is observed on one side, and where on the other side convective mass transfer is observed.

The solution to the problem (1)-(4) can be found by the operational method:

$$\begin{aligned}y(x,\tau)=y_0+\frac{1}{2}\frac{kPR}{\delta D}+\frac{\beta(g-f)}{D^2}\left(\frac{1}{4}R^3+\frac{1}{6}x^3+\frac{1}{6}xR^2\right)+\\+\tau\left[\frac{kP}{\delta R}+\beta(g-f)\left(\frac{3}{2}\frac{R}{D}+\frac{x}{D}\right)\right]+\frac{\tau^2\beta(g-f)}{R}+\\+\sum_{n=1}^{\infty}\frac{R\left[R^2\beta\delta(g-f)(-1)^n-kPD\mu_n^2\right]\cos\mu_n\frac{x}{R}}{\delta D^2\mu_n^4(-1)^n}\exp\left(-\mu_n^2\frac{D\tau}{R^2}\right).\end{aligned} \tag{6}$$

In this solution, $\mu_n = n\pi$, and $0 < \tau < y_0/f$ or $0 < \tau < y_0/g$, where $g > f$.

In posing the problem, a linear change of the concentration versus time at the boundary layer and in the flow core is adopted for simplicity. This dependence is reflected in the boundary condition (3). Averaging over the abscissa leads to the average concentration over the boundary layer thickness

$$\bar{y}(\tau)=y_0+\frac{kPR}{2\delta D}+\frac{3}{8}\beta(g-f)\frac{R^3}{D^2}+\tau\left[\frac{kP}{\delta R}+2\beta\frac{R}{D}(g-f)\right]+\frac{\tau^2\beta(g-f)}{R}. \tag{7}$$

If the retained material is completely isolated (k → 0)

$$\bar{y}(\tau)=y_0+\frac{3}{8}\beta(g-f)\frac{R^3}{D^2}+\frac{2\tau\beta R(g-f)}{D}+\frac{\tau^2\beta(g-f)}{R}. \tag{8}$$

Another formulation of the problem is possible in terms of the permeate. In this case the material separation can be presented as follows:

1) The permeate is removed from the boundary layer through the membrane which is permeable to it;

2) The permeate concentration in the flow between membranes at any moment in time is larger than the concentration in the boundary layer; thus a difference arises in the concentration between the surface of the boundary layer x = 0 and the mixture flow (the mass flow is directed from the membrane space to the boundary layer);

3) In the diffusion boundary layer ($0 \le x \le R$) the permeate is transported towards the membrane under the action of the concentration gradient;

4) The liquid filters through the membrane according to Darcy's law, as in the first case.

The mathematical formulation of the problem is written as follows:

$$\frac{\partial c(x,\tau)}{\partial\tau}=D_{\mathrm{p}}\frac{\partial^2 c(x,\tau)}{\partial x^2}\quad(\tau>0,\ 0<x<R), \tag{9}$$

$$c(x, 0) = c_0, \tag{10}$$

$$-D\frac{\partial c(0, \tau)}{\partial x} - \beta_n[(c_0 - a\tau) - (c_0 - b\tau)] = 0, \tag{11}$$

$$-D\frac{\partial c(R, \tau)}{\partial x} - \frac{k_\mathrm{p}P}{\delta} = 0. \tag{12}$$

The solution is

$$\begin{aligned} c(x, \tau) = {} & c_0 + \frac{\beta_\mathrm{p}(b-a)}{D_\mathrm{p}^2}\left(\frac{R^3}{4} - x^3 - xR^2\right) - \frac{k_\mathrm{p}PR}{2\delta D_\mathrm{p}} + \\ & + \tau\left[\frac{\beta_\mathrm{p}(b-a)}{D_\mathrm{p}}(R-x) - \frac{k_\mathrm{p}P}{\delta R}\right] + \frac{\tau^2\beta_\mathrm{p}(b-a)}{2R} + \\ & + \sum_{n=1}^{\infty}\frac{R\left[k_\mathrm{p}PD_\mathrm{p}\mu_n^2 + R^2\beta_\mathrm{p}\delta(b-a)(-1)^n\right]\cos\mu_n\frac{x}{R}}{\delta D^2\mu_n^4(-1)^n}\exp\left(-\mu_n^2\frac{D_\mathrm{p}\tau}{R^2}\right). \end{aligned} \tag{13}$$

In this solution $\mu_n = n\pi$, and $0 < \tau < c_0/a$ or $0 < \tau < c_0/b$, where $b > a$.

Averaging leads to

$$\bar{c}(\tau) = c_0 + \frac{\beta_\mathrm{p}(b-a)R^3}{2D_\mathrm{p}^2} - \frac{k_\mathrm{p}PR}{2\delta D_\mathrm{p}} + \tau\left[\frac{\beta_\mathrm{p}(b-a)R}{2D_\mathrm{p}} - \frac{k_\mathrm{p}P}{\delta R}\right] + \frac{\tau^2\beta(b-a)}{2R}. \tag{14}$$

Analysis of (8) and (14) shows that the average concentration in the layer varies according to a parabolic law; that is, the overall concentration change of the retained material is described by the equation

$$\bar{y}(\tau) = A_1 + B_1\tau + C_1\tau^2, \tag{15}$$

while for the permeate concentration

$$\bar{c}(\tau) = A_2 + B_2\tau + C_2\tau^2. \tag{16}$$

In these equations $A_1 > 0$, $B_1 > 0$, and $C_1 > 0$. Because the problem is posed such that $\bar{c}(\tau)$ decreases as τ increases, we must have $B_2 < 0$ and $A_2 > 0$. The case $A_2 < 0$ is impossible, because then either $\bar{c}(\tau) < 0$ or $\bar{c}(\tau)$ increases with increasing τ.

For a sufficiently large τ, the value of $\bar{y}(\tau)$ can be approximated as the concentration of the retained material in the concentrate.

Figure 2 shows a laboratory unit with a flat membrane and one cell with a filtration area of 0.02 m^2, on which experiments were conducted on the concentration of phospholipids in sunflower oil. The unit includes a 3-liter tank for oil with a conical bottom and a controlled electrical heater, a 1 m^3/h feed pump, and a manometer and a thermometer to monitor the pressure and temperature of the oil which is being filtered.

Pressed unrefined sunflower oil with an acid number of 1.5 mg KOH and with a phospholipid content of 0.065% based on P_2O_5 was used for the investigations. MFFK-3 brand fluoroplastic [similar to Kel-F] membrane with a 0.45-μm pore diameter was used for separating the phospholipid—triglyceride ystem. The tests were conducted at an oil temperature of 50°C, a pressure of 0.15 mPa and a 1 m/sec flow velocity over the membrane surface. Before the microfiltration, the original oil was processed with 2% water by weight at 50°C for 10 min to transform the phospholipids into the hydrate form. The initial volume of the phospholipid—triglyceride system was 2 liters. A sample was taken every 20 min to determine the phospholipid content. At the time the sample was taken, the filtrate volume was measured to determine the material balance. The test was conducted in the concentration regime. The test lasted 2 h, which corresponded to a 6-fold concentration.

The phospholipid concentration in the filtrates was measured by a calorimetric method. The phospholipid concentration in the concentrate was calculated from the material balance.

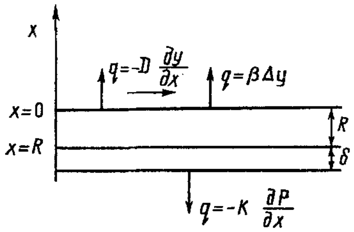

Fig. 1. Diagram of the boundary layer in a membrane channel.

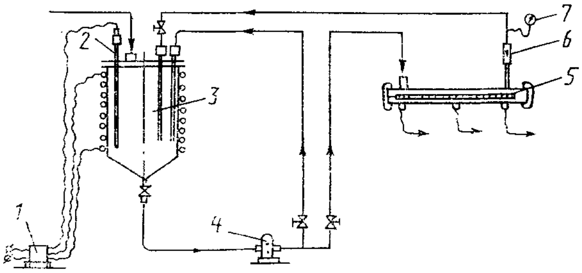

Fig. 2. Laboratory unit with a flat-framed membrane: 1) MKU-48 relay; 2) contact thermometer; 3) tank with electric heater; 4) pump; 5) flat-framed membrane module; 6) float-type flow meter; 7) manometer.

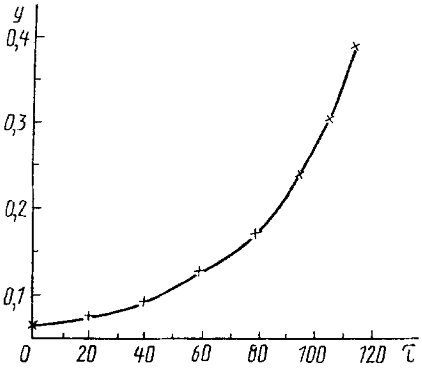

Fig. 3. Phospholipid concentration in the concentrate (y, %, based on P_2O_5) as a function of separation time (τ, min).

Figure 3 shows the phospholipid concentration of sunflower oil in the concentrate as a function of time, which is described by a parabolic law, that is, (15). Thus, the theoretical law of the change in the retained material versus time was confirmed experimentally.

For the vertex of the parabola it is easy to obtain

$$-\frac{R^2}{D} = \tau_1, \tag{17}$$

$$y_0 + \frac{3}{8}\beta(g-f)\frac{R^3}{D^2} - \frac{4\beta^2 R^2 (g-f)^2}{D^2} = y_1. \tag{18}$$

Using (8) for various points on the parabola and (17) and (18) gives

$$\frac{\bar{y}_{n+2} - \bar{y}_{n+1}}{\bar{y}_{n+1} - \bar{y}_n} = \frac{(\tau_{n+2}^2 - \tau_{n+1}^2) - 2\tau_1(\tau_{n+2} - \tau_{n+1})}{(\tau_{n+1}^2 - \tau_n^2) - 2\tau_1(\tau_{n+1} - \tau_n)}. \tag{19}$$

This solution can be used to calculate the degree of concentration for baromembrane separation of solutions as a functon of time, the properties of the solutions, the membrane characteristics, and operational parameters.

NOTATION

y, variable concentration of the material retained by the membrane; y_0, initial concentration of the retained material; D, diffusion coefficient of the retained material; β, mass extraction coefficient; x, coordinate; τ, time; R, boundary layer thickness; k, filtration coefficient through the membrane for the retained material; P, pressure; δ, membrane thickness; f and g, coefficients; c, variable permeate concentration; c_0, initial permeate concentration; D_p, β_p, and k_p, corresponding coefficients of diffusion, mass extraction, and filtration for the permeate; a and b, coefficients; n, index number of points on parabola.

LITERATURE CITED

1. D. R. Trettin and M. Doshi, Ind. Eng. Chem. Fund., 19, 189-194 (1980).
2. D. R. Trettin and M. Doshi, Chem. Eng. Commun., 17, 37-49 (1980).
3. V. L. Vilher and C. K. Colton, Am. J. Chem. Eng., 27, 635-645 (1981).
4. W. Leung and R. F. Probstein, Ind. Eng. Chem. Fund., 18, No. 3, 15-24 (1979).
5. Y. Shen and R. F. Probstein, Ind. Eng. Chem. Fund., 16, 459-464 (1977).
6. R. F. Probstein and W. Leung, J. Phys. Chem., 83, 1228-1231 (1979).
7. A. S. Michaels, Chem. Eng. Progr., 12, 31-43 (1968).
8. R. F. Probstein and Y. S. Shen, Desalination, 24, No. 2, 97-105 (1978).

THERMOPHYSICAL PROPERTIES OF GRANULAR—FIBROUS MATERIAL ON THE TEMPERATURE INTERVAL FROM 175 TO 450 K

Kh. S. Nurmukhamedov, Z. S. Salimov,
S. K. Nigmadzhanov, A. M. Sagitov,
and Kh. A. Khairidinov

UDC 536.2.022-536.63.1

The effective thermal conductivity, specific heat and thermal diffusivity of cottonseed on the temperature interval 175-450 K are calculated for fibrosities and moisture contents varying from 0 to 35%.

The use of both fluidization and hydrodynamically active high-temperature jets for drying granular—fibrous materials requires the determination of the thermophysical properties of the material being dried over a broad range of temperatures [1]. The thermal conductivities, thermal diffusivities and specific heats of granular—fibrous materials have not been sufficiently studied [2, 3]. To a large extent, this is due to the complex multilayer structure of such materials, in particular cottonseed which consists of kernel, cortex and cotton fiber. The few published data mainly relate to the thermophysical properties of bulk cottonseed [2, 3] over a narrow temperature range $T = 294\text{-}359$ K [3]. At the same time, the geometric dimensions and thermophysical properties of the layers differ sharply [4, 5], and between the cortex and the kernel there is a layer of air $(0.05\text{-}0.2)\cdot 10^{-3}$ m thick [6].

A feature of cottonseed is the fibrosity of the outer envelope, which varies from 0 to 35%, commercial seed processed in the oil-extracting industry having a fibrosity of between 4 and 12%. Therefore there is much interest in determining the thermophysical properties of cottonseed with different degrees of fibrosity.

Below, we consider a method of calculating the effective thermal conductivity λ_{eff}, specific heat c_{eff} and thermal diffusivity a_{eff} of granular—fibrous material based on the experimental determination of the thermal conductivities and specific heats c of each of the layers, with reference to cottonseed having various fibrosities, moisture contents and temperatures.

The thermal conductivities and specific heats of the kernel and cortex of the cottonseed were determined on standard IT-λ-400 and IT-s-400 instruments at moisture contents $U = 0.2\text{-}35\%$ and temperatures $T = 175\text{-}450$ K [4, 5].

A. R. Beruni Polytechnic Institute, Tashkent. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 61, No. 6, pp. 958-963, December, 1991. Original article submitted March 15, 1991.

0022-0841/91/6106-1500 $12.50